

EFFECTIVE STRING DESCRIPTION OF THE INTERQUARK POTENTIAL IN THE 3D U(1) LATTICE GAUGE THEORY.¹

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OUTLINE

- ▶ The 3D U(1) lattice gauge model:
 1. Confinement.
 2. Dual formulation and gauge/string duality.
- ▶ Numerical results on the interquark potential: deviations with respect to Nambu-Goto.
- ▶ Conclusions and future directions.

THE $U(1)$ LATTICE GAUGE MODEL IN $3D$

DEFINITION

$$S = \beta \sum_{x \in \Lambda} \sum_{1 \leq \mu < \nu \leq 3} \left[1 - \operatorname{Re} U_{x,\mu} U_{x+a\hat{\mu},\nu} U_{x+a\hat{\nu},\mu}^* U_{x,\nu}^* \right]$$

where Λ is a $3D$ euclidean spacetime lattice and

$$U_{x,\mu} = \exp [ia\vartheta_{\mu}(x + a\hat{\mu}/2)] \in U(1)$$

Since the model is abelian

$$\operatorname{Re} U_{x,\mu} U_{x+a\hat{\mu},\nu} U_{x+a\hat{\nu},\mu}^* U_{x,\nu}^* = \cos(\Delta_{\mu}\vartheta_{x,\nu} - \Delta_{\nu}\vartheta_{x,\mu}) = \cos \vartheta_{x,\mu\nu}$$

Adopting discrete differential forms notation

$$Z = \prod_{c_1} \int_{-\pi}^{\pi} d(\vartheta) e^{-\beta \sum_{c_2} (1 - \cos d\vartheta)}$$

with c_1 and c_2 links and plaquettes on Λ .

THE U(1) LATTICE GAUGE MODEL IN 3D

THE WEAK COUPLING APPROXIMATION

Taking the periodicity of Z into account in the $\beta \gg 1$ approximation

$$Z = Z_{\text{sw}} Z_{\text{top}} = Z_{\text{sw}} \sum_{\{q\}} e^{-2\pi^2\beta(q, \Delta^{-1}q)}$$

where Z_{top} describes a coulomb like gas of topological excitations, Z_{sw} describes spin-waves.

- ▶ The model is always confining in $3D^2$
- ▶ In the semiclassical approximation

$$m_0 = c_0 \sqrt{8\pi^2\beta} e^{-\pi^2\beta v(0)}, \quad \sigma \geq \frac{c_\sigma}{\sqrt{2\pi^2\beta}} e^{-\pi^2\beta v(0)}, \quad v(0) = 0.2527$$

the bounds are saturated and $c_\sigma = 8$, $c_0 = 1$.

- ▶ The ratio

$$\frac{m_0}{\sqrt{\sigma}} = \frac{2\pi c_0}{\sqrt{c_\sigma}} (2\pi\beta)^{3/4} e^{-\pi^2 v(0)\beta/2},$$

can be tuned at will by an appropriate choice of β , in contrast to the general Yang-Mills case.

THE $U(1)$ LATTICE GAUGE MODEL IN $3D$

THE DUAL FORMULATION OF THE MODEL

The dual model is a globally \mathbb{Z} symmetric **spin** model³

$$Z = \sum_{\{*\!l=-\infty\}}^{\{\infty\}} \prod_{*c_1} I_{|d*\!l|}(\beta),$$

where

- ▶ I_α Bessel functions of order α
- ▶ $*c_1$ are links of the dual lattice $*\Lambda$.
- ▶ $*l$ is an integer valued scalar field, and $d*\!l$ differences at neighboring dual sites.

The advantage is twofold:

- ▶ Physical insight into the confinement mechanism : dual superconductor scenario.
- ▶ Ease in numerical computation.

³(Savit, 1980)

THE U(1) LATTICE GAUGE MODEL IN 3D

THE CONFINEMENT MECHANISM AND GAUGE/STRING DUALITY

The dual superconductor scenario of confinement⁴:

- ▶ Condensation of magnetic monopoles drives confinement of electric charges.
- ▶ The dynamics of flux tubes should be described by string like degrees of freedom: no proven gauge/string duality in the general case.

In the U(1) LGT, however, an heuristic proof exists⁵

$$S_{\text{Pol}} = c_1 e^2 m_0 \underbrace{\int d^2\xi \sqrt{g}}_{\text{NG}} + c_2 \frac{e^2}{m_0} \underbrace{\int d^2\xi \sqrt{g} K^2}_{\text{Rigidity}}$$

where c_1 and c_2 are two undetermined constants.

- ▶ At tree level, the rigidity term doesn't contribute to the interquark potential.
- ▶ If $c_1 = \sigma$ and $c_2 = \alpha$ then

$$\sqrt{\sigma/\alpha} = m \sim m_0 .$$

and the rigidity correction is dominant in the $\beta \rightarrow \infty$ limit.

⁴(Polyakov, 1977)

⁵(Antonov, 1998, Polyakov, 1997)

THE U(1) LATTICE GAUGE MODEL IN 3D

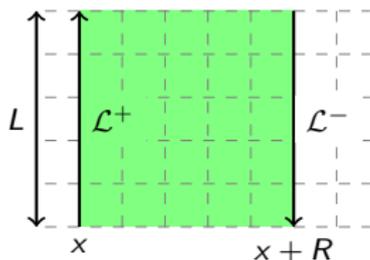
INCLUSION OF POLYAKOV LINES IN THE PARTITION FUNCTION

The interquark potential $V(R)$ can be extracted from

$$G(R) = \langle P^*(R)P(0) \rangle = e^{-N_t V(R)} \propto \int [DX] e^{-S_{\text{eff}}[X]}$$

where S_{eff} is the effective string action and $P(x)$ Polyakov lines.

In the dual formulation, Polyakov lines $P(x)$ are easily included in Z



$$Z_R = e^{-\beta N_l} \sum_{\{^*l = -\infty\}^{\infty}} \prod_{^*c_1} I_{|d^*l + ^*n|}(\beta)$$

where *n is integer valued and nonvanishing only on links dual to a surface bounded by the lines.

Thus in the dual formulation

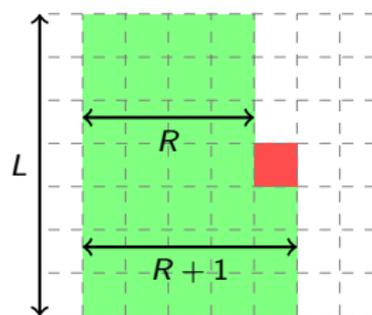
$$G(R) = \frac{Z_R}{Z}$$

which, however, is hard to measure because of an exponentially decaying signal-to-noise ratio.

SIMULATION OF THE THE U(1) LGT IN 3D

SNAKE ALGORITHM AND HIERARCHICAL UPDATE

The problem can be circumvented using the **snake algorithm**⁶:



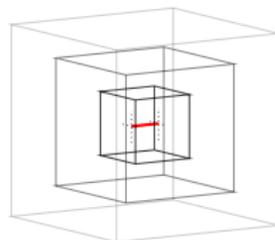
$$\frac{G(R+1)}{G(R)} = \frac{Z_{R+1}}{Z_R} = \frac{Z_{R+1}}{Z_R^{L_d-1}} \frac{Z_R^{L_d-1}}{Z_R^{L_d-2}} \cdots \frac{Z_R^1}{Z_R}$$

where

$$\frac{Z_R^{L_d-i+1}}{Z_R^{L_d-i}} = \left\langle \frac{I_{|d^*l+1}(\beta)}{I_{|d^*l}(\beta)} \right\rangle_{R, L_d-i}$$

are L_d independent local observables.

And efficiency of the computation can be improved by updating the lattice **hierarchically** around the local observable



⁶(de Forcrand, D'Elia, and Pepe (2001))

SIMULATION OF THE U(1) LGT IN 3D

THE GENERAL SETTING AND THE MEASURED QUANTITY

We obtained high precision estimates of

$$Q(R) = -\frac{1}{N_t} \log \frac{G(R+1)}{G(R)} = V(R+1) - V(R)$$

- ▶ The dual model was simulated at several values of β on lattices $L^2 \times N_t$ chosen to avoid finite size effects:

$$N_t, L = \begin{cases} 64a, & \text{for } \beta < 2.4 \\ 128a, & \text{for } \beta \geq 2.4 \end{cases}$$

- ▶ $Q(R)$ was probed in the range $1/\sqrt{\sigma} < R < L/2$
- ▶ A single site metropolis update algorithm was used with Jackknife error estimation.

SIMULATION OF THE U(1) LGT IN 3D

PRELIMINARY MEASUREMENTS

The data was fitted **asymptotically** with

$$V_{NG}(R) = \sigma R \sqrt{1 - \frac{\pi}{12\sigma R^2}}$$

using σ as free parameter in the range $[R_{min}a, La/2]$.

β	σa^2	L, N_t	$1/\sqrt{\sigma}$	$R_{min}\sqrt{\sigma}$
1.7	0.122764(2)	64	$3a$	$11a$
1.9	0.066824(6)	64	$4a$	$17a$
2.0	0.049364(2)	64	$5a$	$20a$
2.2	0.027322(2)	64	$6a$	$26a$
2.4	0.015456(7)	128	$8a$	$34a$

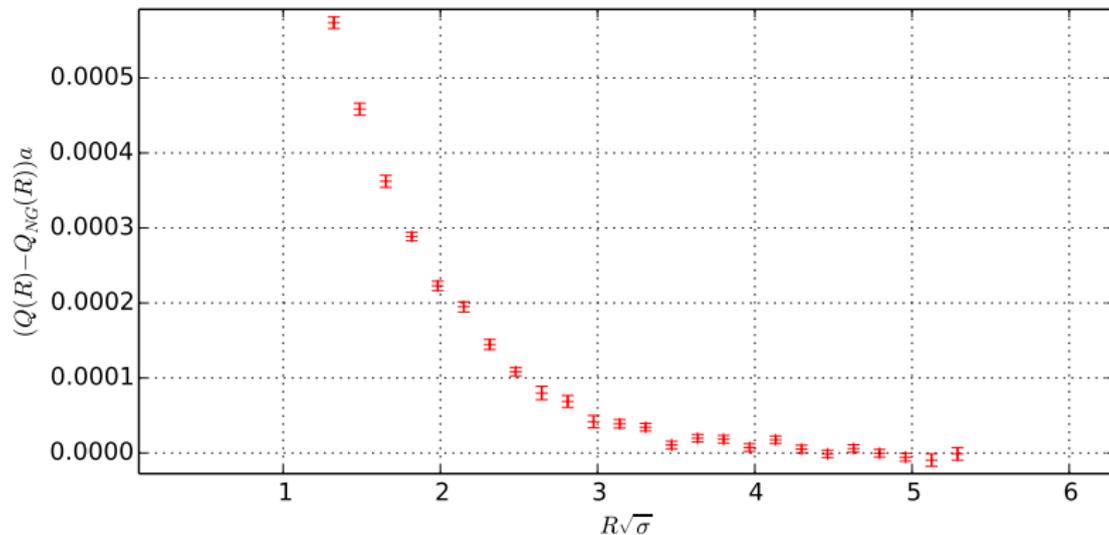
- ▶ At low β , NG describes the data for a wide range of Ra
- ▶ As β grows, the deviations from NG grow: at $\beta = 2.2$ only 6 degrees of freedom can be fitted!

Deviations should be detectable in the range $[a/\sqrt{\sigma}, R_{min}a]$

SIMULATION OF THE U(1) LGT IN 3D

DEVIATIONS WITH RESPECT TO NG

Deviations $(Q(R) - Q_{NG}(R))a$ with respect to NG at $\beta = 2.2$ on a $(64a)^3$ lattice.



The best fit value of $\sigma a^2 = 0.027322(2)$ was obtained with $R_{min}\sqrt{\sigma} = 4.3$.

DEVIATIONS WITH RESPECT TO NG

HOW TO EXPLAIN THEM?

In general

$$S_{eff} = S_{NG} + S_b + S_{2,K}$$

Up to the resolution of our data

$$S_{NG} \simeq S_{cl.} + \frac{\sigma}{2} \int d^2\xi \left[\partial_\alpha X \cdot \partial^\alpha X - \frac{1}{4} (\partial_\alpha X \cdot \partial^\alpha X)^2 \right],$$

$$S_b \simeq b_2 \int d\xi_0 [\partial_1 \partial_0 X \cdot \partial_1 \partial_0 X],$$

$$S_{2,K} \simeq \alpha \int (\Delta X)^2$$

For each we can compute the L.O. contribution to $V(R)$ perturbatively⁷

$$V_b(R) = -b_2 \frac{\pi^3}{60} \frac{1}{R^4}, \quad V_r(R) = -\frac{m}{2\pi} \sum_{n=1}^{\infty} \frac{K_1(2nmR)}{n}, \quad m = \sqrt{\frac{\sigma}{2\alpha}}$$

⁷(Aharony and Field, 2011, Billó et al., 2012, Klassen and Melzer, 1991, Nesterenko and Pirozhenko, 1997) 

DEVIATIONS WITH RESPECT TO NG

RIGIDITY AND BOUNDARY AT LO

The boundary correction V_b alone can't describe the deviations:

- ▶ $\chi_R^2 \sim 1$ only for very large values of $R_{\min}\sqrt{\sigma}$.
- ▶ The best fit values of b_2 have the wrong scaling behaviour:

$$b_2\sigma^{3/2} = 0.033(3), \quad \beta = 1.7$$

$$b_2\sigma^{3/2} = 0.62(6), \quad \beta = 2.4$$

- ▶ A complementary test with the potential

$$V(R) = \frac{A}{R^B}$$

with A, B free parameters shows that $b \neq 4$.

Fitting with the rigidity correction V_r works much better:

- ▶ Good fits are obtained already at small distances:

$$ma = 0.112(2), \quad \chi_r^2 = 1.03, \quad R_{\min}\sqrt{\sigma} = 2.15$$

to be compared with $R_{\min}\sqrt{\sigma} = 4.3$ for NG.

- ▶ The best fit value of m scales with m_0 .

DEVIATIONS WITH RESPECT TO NG

RIGIDITY AT NLO

The NLO correction due to the rigidity contribution can be computed in the large D limit⁸

$$V_2(R) = - \left(\frac{\pi D}{24} \right)^2 \frac{3}{20m\sigma R^4}$$

and in the general case⁹

$$V_2'(R) = -(D-2)(D-10) \left(\frac{\pi}{24} \right)^2 \frac{3}{20m\sigma R^4}$$

- ▶ This contribution is detected within the precision of our data and contributes to the best fit value of ma .
- ▶ It is entangled to the boundary correction, which then cannot be neglected!

⁸(Braaten et al., 1987)

⁹(German and Kleinert, 1989)

DEVIATIONS WITH RESPECT TO NG

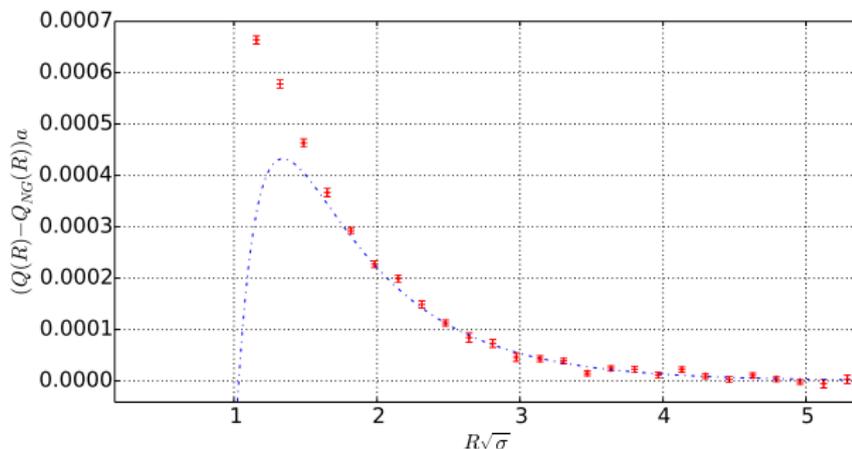
3 PARAMETERS FIT OF THE DATA

$$V(R) = V_{NG}(R) + V_r(R) + V_2'(R) + V_b(R)$$

using σ , m and b_2 as free parameters results in the best fit values

$$\sigma a^2 = 0.027318(2), \quad ma = 0.11(1), \quad b_2 \sigma^{3/2} = 0.005(1),$$

with $\chi_r^2 = 1.2$ and $R_{min} \sqrt{\sigma} = 1.65$.



In the plot: The deviations $(Q(R) - Q_{NG}(R))a$ and the curve $Q_r(R) + Q_2'(R) + Q_b(R)$ calculated with the best fit values for σ , m and b_2 .

DETERMINATION OF ma

The same analysis for the other couplings leads to:

β	ma	$m_0 a$	m/m_0
1.7	0.28(9)	0.88(1)	0.32(10)
1.9	0.25(4)	0.56(1)	0.45(7)
2.0	0.17(2)	0.44(1)	0.39(4)
2.2	0.11(1)	0.27(1)	0.41(4)
2.4	0.06(2)	0.20(1)	0.30(10)

- ▶ Takes into account the interplay between σ , m , and b_2 in the error.
- ▶ m scales with m_0 as predicted by Polyakov.

Our estimate of the rigidity parameter is

$$m/m_0 = 0.35(10) .$$

CONCLUSIONS

- ▶ The strong deviations with respect to NG observed in the U(1) LGT in $3D$ can be explained by the addition of a rigidity term to the effective string action, as predicted by Polyakov. This contribution becomes dominant in the limit $\beta \rightarrow \infty$.
- ▶ Future directions:
 1. Try to disentangle the NLO rigidity contribution from the boundary correction.
 2. Study the behaviour intrinsic width of the string and compare with predictions of the string with rigidity.
 3. Finite temperature behaviour of the interquark potential.

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